Hdiv and Finite Elements

NeoPZ

- Hp-Adaptive 3D
  - H1 approximations (1987)
  - DG approximations
  - Hdiv approximations
  - Hcurl approximations
  - SBFem approximations
- Hybridization
- Nonlinear geometric maps (exact bc representation)
- H-refinement patterns
- Substructuring
- Multiscale approximations

- Error estimation adaptivity
  - H1, projecting on Hdiv
  - Hdiv, projecting on H1
  - Hybrid H1, two projections
  - MHM, skeleton refinement
- Hdiv
  - Innovative way of generating Hdiv spaces (2D, 3D)
  - Enhanced accuracy of the primal variable
  - Divergence free space
  - Hdiv constant spaces
  - Mixed elasticity 2D and 3D
- Iterative methods
  - Very effient iterative solver for Hdiv approximations
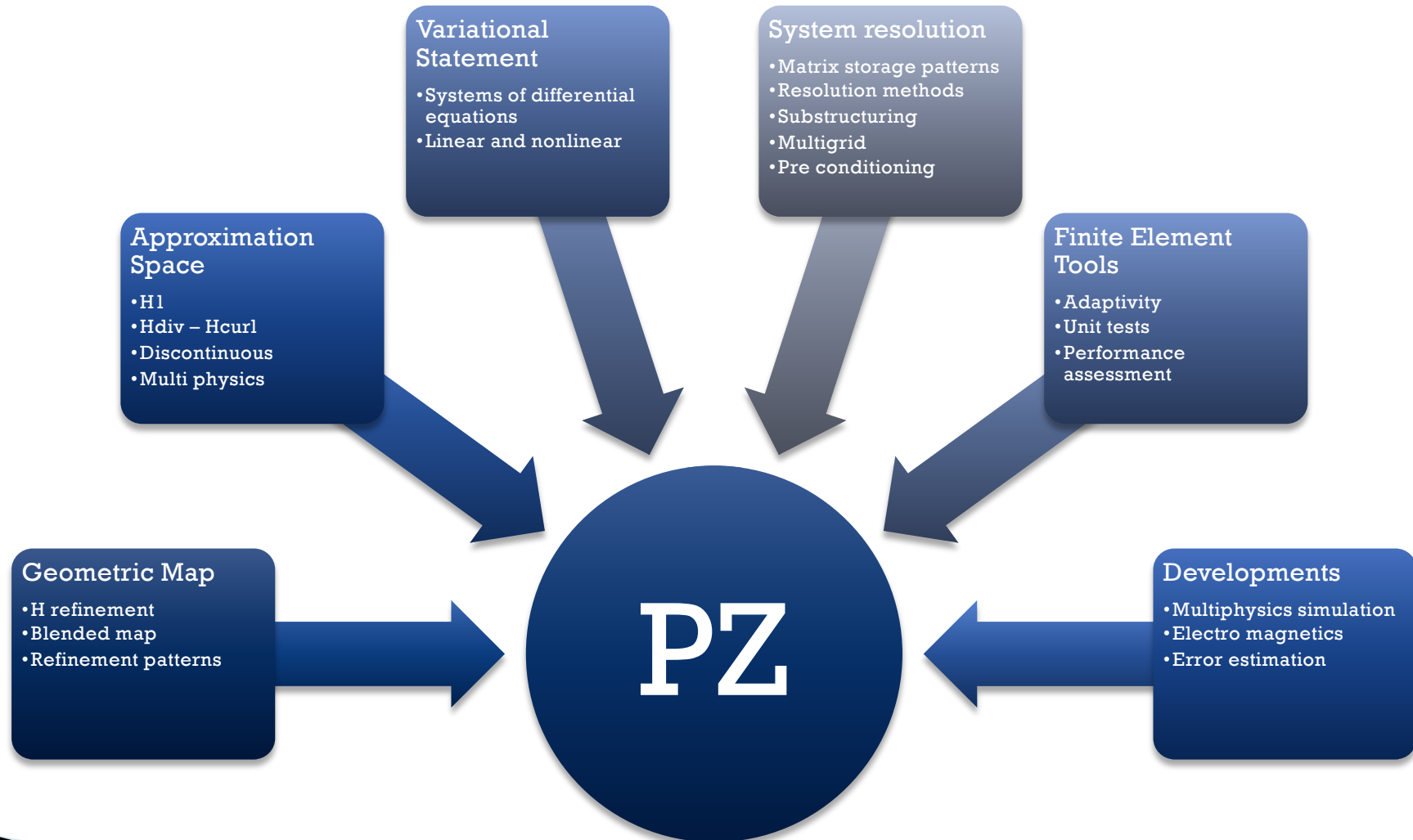 - Dorhmann preconditioner for substructured meshes
  - Saddle point problems

Applied FEM Research at LabMeC

- Multiphysics approximations
  - Combination of De Rham compatible spaces
  - Simulation of coupled fenomena
- Reservoir simulator
  - Fluid flow - Hdiv
  - Pressure – L2
  - Elasticity – H1
  - Transport – DG
  - Constitutive model using average pressure
- Simulation of fractured porous media
  - Coupling of 3D and 2D Hdiv approximations
  - Advanced mesh generation
  - Statistics of fluid transmission between fractures

- Hydraulic fracture propagation
  - Coupling of fluid flow in fractures with elastic response
  - Simulation of fracture propagation (Propag simulator Petrobras)
  - Use of reduced approximation spaces
- Coupled Stokes-Darcy flow
  - Pointwise conservative Stokes simulator based on hybridized Hdiv approximations
  - Straightforward coupling of Stokes-Darcy using a unique Hdiv space
  - Multiscale approximation of Stokes flow
  - Extension of Stokes simulation to incompressible Navier Stokes (Re 4000)
- Simulation of wave-guide problems
  - Impact of waveguide transitions
  - Use 2D eigenmode analysis to lower cost of 3D simulation

Applied FEM Research at LabMeC

- Simulation of wellbore stability
  - Simulate elastoplastic response of the well during drilling
  - Hp-adaptivity applied to elasto-plasticity
- SBFem approximations
  - Formal proof of convergence of SBFem
  - SBFem applied to 2D and 3D Darcy and elasticity
  - Extension of SBFem to Hdiv approximations

- Coupling of 1D wellbore flow with 3D flow in horizontal wells
  - hp-adaptivity
  - nonlinear geometric maps
  - Substructuring to separate linear and nonlinear flow
  - IP3D software used by Petrobras
- Multiscale Hybrid Mixed simulations (MHM)
  - MHM-H1 and MHM-Hdiv in 2D and 3D
  - MHM for Darcy and Elasticity
  - MHM for fractured porous media
  - MHM applied to polygonal domains
  - Error estimation and adaptivity for MHM

Global structure of NeoPZ

UNICAMP

Variational Statement
- Systems of differential equations
- Linear and nonlinear

System resolution
- Matrix storage patterns
- Resolution methods
- Substructuring
- Multigrid
- Pre conditioning

Approximation Space
- H1
- Hdiv – Hcurl
- Discontinuous
- Multi physics

Finite Element Tools
- Adaptivity
- Unit tests
- Performance assessment

Geometric Map
- H refinement
- Blended map
- Refinement patterns

Developments
- Multiphysics simulation
- Electro magnetics
- Error estimation

PZ

https://github.com/labmec/neopz

GitHub

LabMec

Summary

- NeoPZ is an object oriented general purpose finite element library

- A very efficiently structured FEM library allows to aggregate graduate research results

- NeoPZ is used to develop innovative finite element algorithms applied to problems relevant to industry

Acknowledgement