POD-assisted computations of incompressible fluid flows: applications to marine energy

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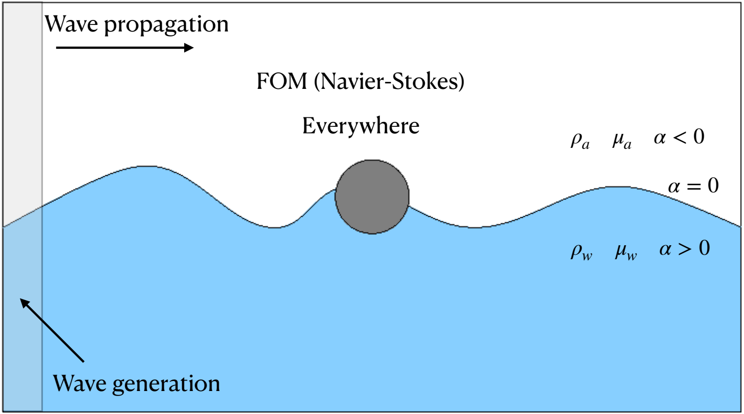
General problem and flow configuration

Wave energy converters

Source: Politecnico di Torino & Marine Offshore renewable energy

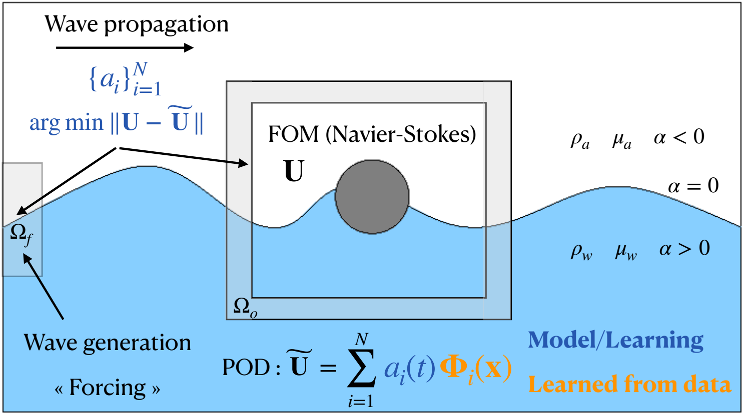
Multi-fidelity modeling: FOM + Proper Orthogonal Decomposition



↪ **Incompressible bi-fluid Navier-Stokes equations (to avoid surface fitted-grids)**

↪ **Too costly! ⇒ we can only afford few numerical simulations! (how to select??)**

Multi-fidelity modeling: FOM + Proper Orthogonal Decomposition



$\hookrightarrow$ The POD basis functions $\{\mathbf{\Phi}_i\}_{i=1}^{N}$ are **learned from data** (previous simulations)

$\hookrightarrow$ The POD coefficients $\{a_i\}_{i=1}^{N}$ can be obtained by **optimization (Galerkin-free)**

Reduced Order Model

Generalized coordinates $\{a\}_{i=1}^{N_r}$

Galerkin-free Reduced Order Model

What variables? $\Rightarrow$ whose are measured at inflow AND required for FOM BCs

$$\text{Velocity:} \quad \widetilde{\boldsymbol{u}} = \boldsymbol{u}_g + \sum_{i=1}^{N_r} \hat{u}_i \boldsymbol{\Phi}_i,$$

$$\text{Color function (VOF, LS):} \quad \widetilde{\alpha} = \alpha_g + \sum_{i=1}^{N_r} \hat{\alpha}_i \Psi_i \quad \Rightarrow \ \rho,\ \mu.$$

The functions $\boldsymbol{u}_g$ and α_g can be snapshots average, or any desired functions

$\hookrightarrow$ $\{\hat{u}\}_{i=1}^{N_r} \leftarrow$ Least squares minimization of $\|\boldsymbol{u}_h - \widetilde{\boldsymbol{u}}\|_2$ in "gray" domains $\Omega_o \cup \Omega_f$,

$\hookrightarrow$ $\{\hat{\alpha}\}_{i=1}^{N_r} \leftarrow$ Least squares minimization of $\|\alpha_h - \widetilde{\alpha}\|_2$ in "gray" domains $\Omega_o \cup \Omega_f$.

$\hookrightarrow$ More stable than classical Galerkin projection since HD informations are involved

In any case, an adapted POD subspace is required!!

Example for sea wave energy converter (point absorber)

In sample ("reproduction" with $N_r = 30$ modes)
POD basis Φ built using snapshots from "exact wave"
Very good approximation with speedup ≈ 10

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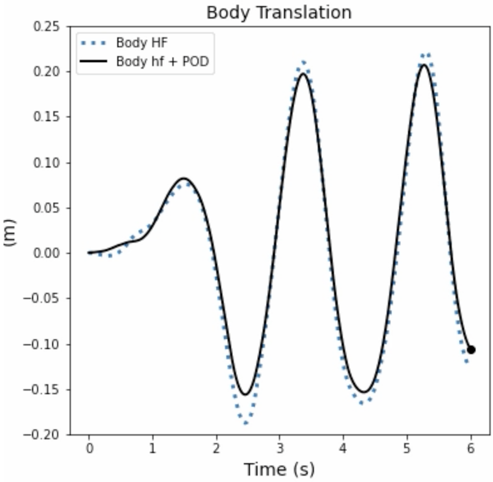
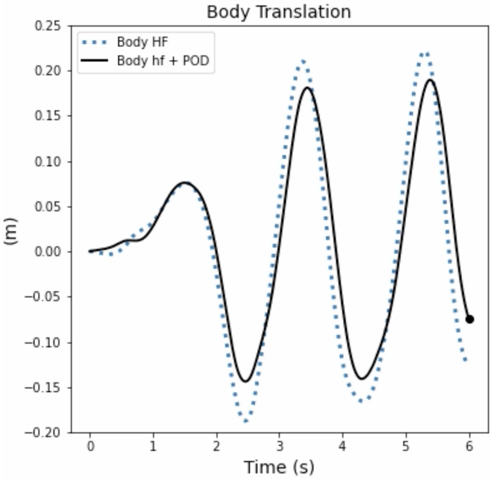


Out-of-sample ("prediction" with $N_r = 30$ modes)
POD basis Φ built using snapshots from two "nearby" waves (in parameter space)
Good approximation with speedup ≈ 10

Example for sea wave energy converter (point absorber)



Out-of-sample ("prediction" with $N_r = 30$ modes)
POD basis Φ built using snapshots from two "distant" waves (in parameter space)
Poor approximation with speedup ≈ 10

The scientific barriers to be overcome & ongoing work

- **The scientific barriers**
  - Sea waves: non-linear transport ("moving front")!
    $\hookrightarrow$ POD is an affine approximation with linear subspace $\Rightarrow$ lot of modes are required!!
  - A robust POD basis is required (to be able to accurately approximate a set of waves, not only a single one)

- **Ongoing work**
  - Non linear transport
    Snapshots clustering, then POD for each cluster (piecewise linear approx.)
    Snapshots mapping onto reference solution, and then POD (non linear approx.)
    Optimal transport (non-linear approx.) + quadratic approximation (Stanford)
    Replace POD reduced order model with an asymptotic model like Boussinesq (ongoing work with CARDAMOM Team)
  - Robust POD subspace with "optimal" sampling
    Sampling on the Grassmann manifold ("uniform" sampling in the solution space)